

Closing Tue: 1.2/3, 1.6(pt. 1)
Exam 1 is Thursday!

Section 1.3 & 1.6 - Linear Functions with Business Examples

Entry Task (watch the units):

Try this on your own now!

You sell Things. Your total cost, in **hundreds** of dollars, for producing x **hundred** Things is given by

$$TC(x) = 40 + 0.5x.$$

1. How much do you pay in FC?
2. What is TC if you make 700 Things?
3. How many Things are you making if total costs are \$12,000?

STEP 1 ALWAYS WRITE THIS FIRST

$$x = \underline{\hspace{2cm}} \text{ HUNDRED THINGS}$$

$$TC(x) = \underline{\hspace{2cm}} \text{ HUNDRED DOLLARS}$$

THESE ARE THE LABELS TO USE WHEN TRANSLATING AT THE START OR END OF ANY PROBLEM INVOLVING THIS FUNCTION

$$\boxed{1} \quad FC = TC(0) = \boxed{40 \text{ HUNDRED DOLLARS}} \\ = \$4000$$

$$\boxed{2} \quad 700 \text{ THINGS} \Leftrightarrow x = 7 \text{ HUNDRED THINGS} \\ TC(7) = 40 + 0.5 \cdot 7 = \boxed{43.5 \text{ HUNDRED DOLLARS}} \\ = \$4350$$

$$\boxed{3} \quad \$12000 \Leftrightarrow 120 \text{ HUNDRED DOLLARS}$$

$$TC(x) \stackrel{?}{=} 120 \Rightarrow 40 + 0.5x \stackrel{?}{=} 120$$

$$\Rightarrow 0.5x = 80$$

$$\Rightarrow x = \frac{80}{0.5} = 160$$

$$x = \boxed{160 \text{ HUNDRED THINGS}} \\ = 16000 \text{ THINGS}$$

Linear functions are written as:

$$f(x) = mx + b \quad (\text{slope-intercept})$$

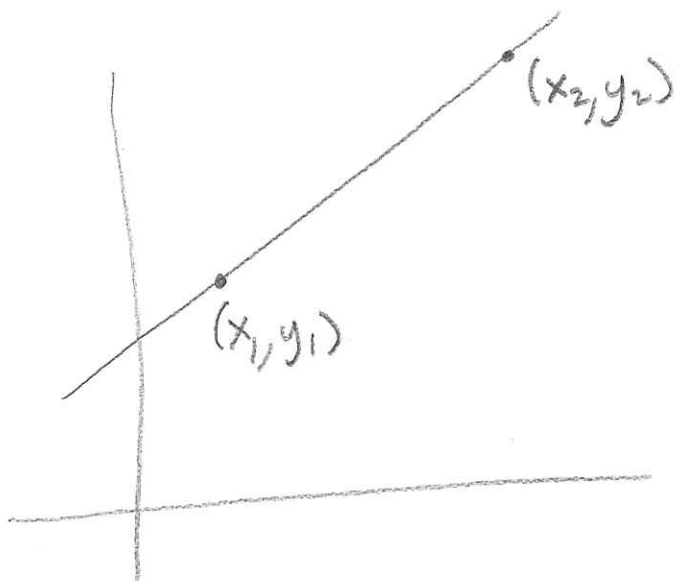
or

$$f(x) = m(x - x_0) + y_0 \quad (\text{point-slope})$$

How to find the equation of a line

Given any two points (x_1, y_1) , (x_2, y_2) :

1. Compute slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2. Write: $y = m(x - x_1) + y_1$



Examples:

1. Find the equation of the line that goes through $(1,1)$ and $(5,7)$.

$$\text{SLOPE} = \frac{7-1}{5-1} = \frac{6}{4} = \frac{3}{2} = 1.5$$

$$y = \frac{3}{2}(x-1) + 1$$

OR

$$y = \frac{3}{2}x - \frac{3}{2} + 1$$

$$y = \frac{3}{2}x - \frac{1}{2}$$

NOTE: CAN USE EITHER POINT HERE

$$y = \frac{3}{2}(x-5) + 7$$
$$= \frac{3}{2}x - \frac{15}{2} + 7$$

$$y = \frac{3}{2}x - \frac{1}{2}$$

SAME

2. Find the equation of the line that goes through $(4,1)$ and $(6,15)$.

$$\text{SLOPE} = \frac{15-1}{6-4} = \frac{14}{2} = 7$$

$$y = 7(x-4) + 1$$

$$y = 7x - 28 + 1$$

$$y = 7x - 27$$

OR

$$y = 7(x-6) + 15$$

$$y = 7x - 42 + 15$$

SAME

$$y = 7x - 27$$

CHECK! $x=4, y=1 \Rightarrow 1 \stackrel{?}{=} 7(4) - 27$ YES ✓

$x=6, y=15 \Rightarrow 15 \stackrel{?}{=} 7(6) - 27$ YES ✓

Line Facts:

- Anything that can be written as $ax + by = c$ gives a line.

→ Ex $5x + 2y = 20$
 $2y = 20 - 5x$
 $y = \frac{20}{2} - \frac{5}{2}x = 10 - 2.5x$

SLOPE
↓

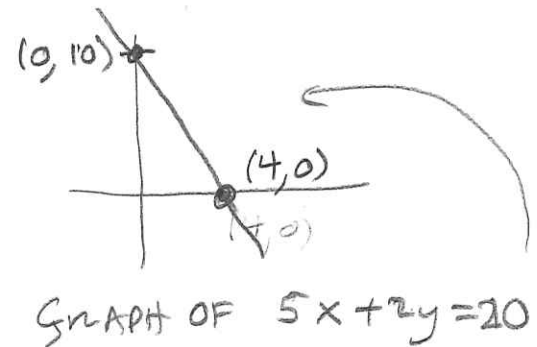
- The *x-intercept* is the x-value where the line crosses the x-axis.
Note: At this point, $y=0$.

→ Ex $5x + 2y = 20$
 ↑
 0
 } $\Rightarrow 5x + 0 = 20$
 } $\Rightarrow \boxed{x = 4}$
 ↑
 x-intercept

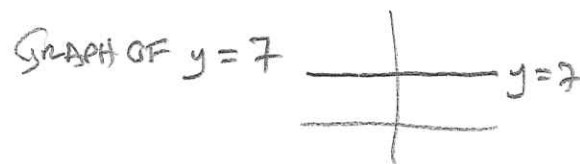
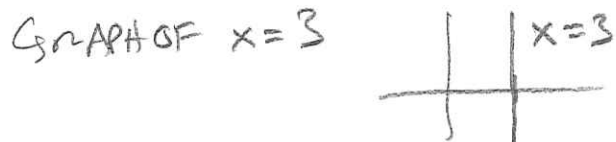
- The *y-intercept* is the y-value where the line crosses the y-axis.
Note: At this point, $x=0$.

Ex $5x + 2y = 20$
 ↑
 0
 } $\Rightarrow 0 + 2y = 20$
 } $\Rightarrow \boxed{y = 10}$
 ↑
 y-intercept

- Two lines are parallel if they have the same slope.



- $y = h$ gives a horizontal line.
- $x = k$ gives a vertical line.



1.3 & 1.6: More Linear Business Apps

Recall: If p = price & $FC = TC(0)$, then

Totals

$$TR(q) = pq,$$

$$TC(q) = FC + VC(q)$$

$$P(q) = TR(q) - TC(q)$$

Overall Averages

$$AR(q) = \frac{TR(q)}{q} = p$$

$$AC(q) = \frac{TC(q)}{q}, AVC(q) = \frac{VC(q)}{q}$$

Marginals

$$MR(q) = \frac{TR(q + \text{'one item'}) - TR(q)}{(q + \text{'one item'}) - q}$$

$$MC(q) = \frac{TC(q + \text{'one item'}) - TC(q)}{(q + \text{'one item'}) - q}$$

Example: You sell Objects. Each object sells for \$5. Your total cost function is linear. When you produce 1 Object, total cost is \$53. When you produce 10 Objects, your total cost is \$80.

(a) TC LINEAR $\Rightarrow TC(x) = m(x - x_1) + y_1$

2 PTS: $(x, y) = (1, 53), (10, 80)$

SLOPE = $m = \frac{80 - 53}{10 - 1} = \frac{27}{9} = 3$

$TC(x) = 3(x - 1) + 53 = 3x - 3 + 53$

• $TC(x) = 3x + 50$ CHECK! $x=1 \Rightarrow y=53 \checkmark$
 $x=10 \Rightarrow y=80 \checkmark$

• $FC = TC(0) = 50$ DOLLARS

• $VC(x) = 3x$

What are the functions for...

- (a) ... TC, FC and VC?
- (b) ... TR and Profit?
- (c) ... MR, MR and MP?
- (d) ... AC and AVC?

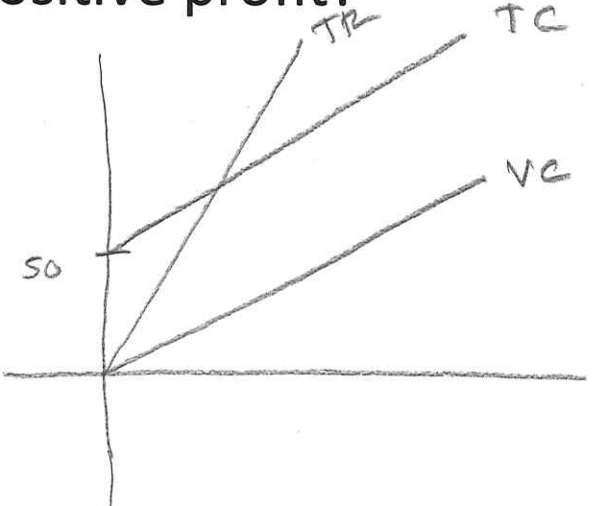
(b) $TR(x) = 5x$

PROFIT = $P(x) = TR(x) - TC(x)$ ← important to distribute here!
 $= (5x) - (3x + 50)$
 $= 5x - 3x - 50$

$P(x) = 2x - 50$

CHECK: $P(0) = -50$ ← ALWAYS NEGATIVE FC!

For what quantities do you make a positive profit?



(c) $MR(x) = TR(x+1) - TR(x) = 5(x+1) - 5x = 5x + 5 - 5x$

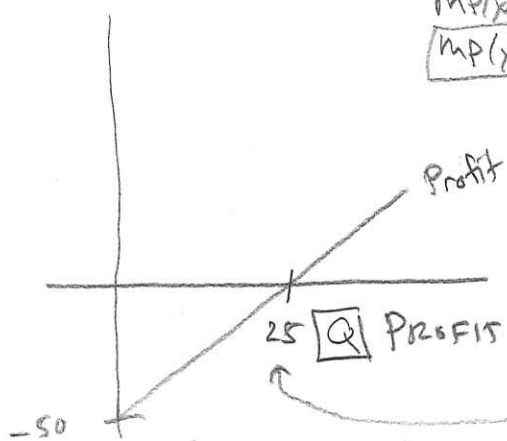
$MR(x) = 5$

$MC(x) = TC(x+1) - TC(x) = (3(x+1) + 50) - (3x + 50)$
 $= 3x + 3 + 50 - 3x - 50 = 3$

$MC(x) = 3$

$MP(x) = MR(x) - MC(x) = 5 - 3 = 2$

$MP(x) = 2$



(d) $AC(x) = \frac{TC(x)}{x} = \frac{3x + 50}{x} = 3 + \frac{50}{x}$
 $AVC(x) = \frac{VC(x)}{x} = \frac{3x}{x} = 3$

$25 \text{ Q} \text{ PROFIT} > 0 \Leftrightarrow 2x - 50 > 0 \Leftrightarrow 2x > 50 \Leftrightarrow x > 25$

Midterm 1 is Thursday

- Sup 1-9, 1.1-1.3, 1.6 (pt 1)
- In your normal quiz section.
- Four pages of questions.
- You get 50 minutes.
- Allowed:
 - Ti – 30x IIS calc (*only this model*)
 - 8.5 x 11 inch *handwritten* notes
 - Ruler
 - Something to write with
(No red/green pens)
- Multiple versions of the exam. We report all suspicions of cheating.
- Show/Label your work.

Studying notes:

- There WILL be problems/graphs ***directly*** from homework.
- There WILL be at least two pages of overall graphs where you have to draw lines to compute/use rates. At least one of those will involve TC or VC and the key business applications!
- There WILL be a page that has an incremental graph (like change in temp or MC/AC/AVC, etc.) But your ruler away and correctly read off your answers!
- There WILL be a page with some algebra (things like you are doing in the 1.1-1.3, 1.6(pt 1) HW).
- So be ready for all these things! After you have mastered all the homework, then practice your understanding using exams from the exam archive.